Since, for every τ, we can choose ρ and σ so that $\lambda_{\rho\sigma}{}^{\tau}\neq 0$ and $\lambda_{\rho\sigma}{}^{\tau'}=0$ for $\tau'\neq\tau$, we conclude from (13) that $x_{\alpha\tau}=0$. Thus $[E_\alpha,L_\rho]=0$ and Eq. (1) holds.

The theorem also holds for any compact internal symmetry group.[4] Then, if the group is not semisimple, it is the direct product of a semisimple group and an Abelian group (toroid). The generators of the toroid commute with all the generators of the internal symmetry group, and the proof of Eq. (6) remains unchanged.

Our proof does not exclude the possibility that the internal symmetry group and the Lorentz group are embedded in some larger symmetry group.[5] If this is the case, one must face the problem of interpreting the additional symmetry operations associated with this larger group.

[4] L. Pontryagin, *Topological Groups* (Princeton University Press, Princeton, New Jersey, 1939), p. 282.

[5] For a particular attempt, cf. Ref. 1.

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Consequences of Crossing Symmetry in SU_3†

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The following processes of 2 octets transforming into 2 octets are discussed on the basis of crossing symmetry; $P+B\to P+B$, $B+B\to B+B$, $P+V\to P+V$, $\cdots$ and their crossed channels, where B, P, and V represent baryons, pseudoscalar mesons, and vector mesons, respectively. The relation between channel amplitudes, the number of independent channel amplitudes, and spin selection rules are systematically obtained.

A SET of states that transform into one another under the unitary transformations SU_3 will form multiplets that are labeled by two quantum numbers (λ,μ). In the octet model,[1,2] the baryons $B=(N,\Sigma,\Lambda,\Xi)$, the antibaryons $\bar{B}$, the pseudoscalar mesons $P=(K,\pi,\eta,\bar{K})$, and the vector mesons $V=(K^*,\rho,\phi^0,\bar{K}^*)$ are assigned to the (1,1) representation of the group SU_3. The amplitude $(ab|cd)$ for the reaction $a+b\to c+d$ can be described as two octets transforming into two other octets.

Two octets (1,1) can couple together to form the product representations (2,2), $(1,1)_s$, (0,0), (0,3), (3,0), and $(1,1)_a$. The representation $(1,1)_s$ transforms with a positive phase whereas the representation $(1,1)_a$ transforms with a negative phase under an R transformation[1] that is independent of SU_3. There are thus six channel amplitudes A_{27}, A_{8s}, A_1, $A_{\overline{10}}$, A_{10}, and A_{8a} which are diagonal elements of the S matrix for the representations (2,2), $(1,1)_s$, (0,0), (0,3), (3,0), and $(1,1)_a$, respectively. There are also two nondiagonal channel amplitudes A_{as} and A_{sa} that couple the representations $(1,1)_s$ and $(1,1)_a$.

One can sometimes obtain relations among the channel amplitudes by use of invariance under time reversal, $(ab|cd)\to(cd|ab)$, charge conjugation, $(ab|cd)\to(\bar{a}\bar{b}|\bar{c}\bar{d})$, and parity operation—all of which hold in the strong interactions—together with crossing symmetry. We shall see that most of the relations follow only from time-reversal invariance in the direct channel (channel I).

Let us define the three channels I, II, and III as follows:

$$\text{Channel I:}\quad (ab|cd),\quad \text{amplitude}=A\,;$$
$$\text{Channel II:}\quad (\bar{c}a|d\bar{b}),\quad \text{amplitude}=B\,;$$
$$\text{Channel III:}\quad (\bar{c}b|\bar{a}d),\quad \text{amplitude}=C.$$

Then the amplitudes A, B, and C are related to each other by crossing symmetry; i.e., $A=O_2B$ and $A=O_3C$, where the crossing matrices O_2 and O_3 are[3,4]

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[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); California Institute of Technology Report CTSL-20, 1961 (unpublished).

[2] Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).

[3] From the crossing matrix for $A=O_2B$, one can obtain $A=O_3C$ in the following way:

$$(ab|cd)\to(ba|cd)\overset{O_3}{\to}(\bar{c}a|\bar{b}d)\to(\bar{c}a|d\bar{b}),$$

$$(ab|cd)\overset{O_2}{\to}(\bar{c}a|d\bar{b}).$$

This results in $A_a\to -A_a$, $A_{as}\to -A_{as}$, $C_a\to -C_a$, and $C_{sa}\to -C_{sa}$, where A_a represents $A_{\overline{10}}$, A_{10}, and A_{8a}.

[4] The crossing matrix has been considered by R. E. Cutkosky, Ann. Phys. **23**, 405 (1963); D. E. Neville, Phys. Rev. **132**, 844 (1963); J. J. de Swart, Nuovo Cimento **31**, 420 (1964). We thank S. Okubo and B. Lee for pointing this out. Equation (1) is read as

$$A_{27}=\frac{7}{40}B_{27}+\frac{1}{5}B_{8s}+\frac{1}{8}B_1-\frac{1}{12}B_{\overline{10}}-\frac{1}{12}B_{10}-\frac{1}{3}B_{8a},\ \text{etc.}$$

The implications of time-reversal in elastic scattering in connection with SU_3 have been remarked on by P. G. O. Freund, H. Ruegg, D. Speiser, and A. Morales, Nuovo Cimento **25**, 307 (1962); P. Tarjanne, Ann. Acad. Sci. Fennicae Ser. A VI, 105 (1962).

	(27)	(8s)	(1)	$(\overline{10})$	(10)	(8a)	(as)	(sa)
(27)	$\frac{7}{40}$	$\frac{1}{5}$	$\frac{1}{8}$	$\mp\frac{1}{12}$	$\mp\frac{1}{12}$	$\mp\frac{1}{3}$	0	0
(8s)	$\frac{27}{40}$	$-\frac{3}{10}$	$\frac{1}{8}$	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	$\pm\frac{1}{2}$	0	0
(1)	$\frac{27}{8}$	1	$\frac{1}{8}$	$\pm\frac{5}{4}$	$\pm\frac{5}{4}$	± 1	0	0
$(\overline{10})$	$\mp\frac{9}{40}$	$\mp\frac{2}{5}$	$\pm\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4}$	0	$\mp\frac{1}{\sqrt{5}}$	$\frac{1}{\sqrt{5}}$
(10)	$\mp\frac{9}{40}$	$\mp\frac{2}{5}$	$\pm\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4}$	0	$\pm\frac{1}{\sqrt{5}}$	$-\frac{1}{\sqrt{5}}$
(8a)	$\mp\frac{9}{8}$	$\pm\frac{1}{2}$	$\pm\frac{1}{8}$	0	0	$\frac{1}{2}$	0	0
(as)	0	0	0	$\frac{\sqrt{5}}{4}$	$-\frac{\sqrt{5}}{4}$	0	$\pm\frac{1}{2}$	$\frac{1}{2}$
(sa)	0	0	0	$\mp\frac{\sqrt{5}}{4}$	$\pm\frac{\sqrt{5}}{4}$	0	$\frac{1}{2}$	$\pm\frac{1}{2}$

(1)

the upper and lower signs being referred to O_2 and O_3, respectively. From matrices (1), it follows immediately that

$$A_{as}=A_{sa}\leftrightarrow B_{\overline{10}}=B_{10}\leftrightarrow C_{as}=C_{sa}, \quad (2)$$

$$A_{\overline{10}}=A_{10}\leftrightarrow B_{as}=B_{sa}\leftrightarrow C_{as}=-C_{sa}, \quad (3)$$

$$A_{as}=-A_{sa}\leftrightarrow B_{as}=-B_{sa}\leftrightarrow C_{\overline{10}}=C_{10}, \quad (4)$$

where $\leftrightarrow$ means that one equality implies the other.

(1) $P+B\rightarrow P+B(a=P, b=B, c=P, d=B$ or $a=V, b=B, c=V, d=B)$. Time-reversal invariance in channel I leads to $A_{as}=A_{sa}$ so that Eq. (2) holds and thus there are seven independent amplitudes. The relation $B_{\overline{10}}=B_{10}$ can also be obtained, in general, from invariance under the product of C (charge conjugation) and P (parity operation) in the channel that has nucleon number zero—except for $B+\bar{B}\rightarrow P+V$, in which case there are eight independent amplitudes.

(2) $B+B\rightarrow B+B(a=B, b=B, c=B, d=B)$. Again Eq. (2) follows from time-reversal invariance. In addition, one has $C_{\overline{10}}=C_{10}$ because channels II $(\bar{B}+B\rightarrow B+\bar{B})$ and III $(\bar{B}+B\rightarrow\bar{B}+B)$ are similar except that the final particles appear in a different order, so that Eq. (4) holds. Then, it follows from Eqs. (2) and (4) that

$$A_{as}=A_{sa}=0;\quad B_{\overline{10}}=B_{10},\quad B_{as}=-B_{as};$$
$$C_{as}=C_{sa},\quad C_{\overline{10}}=C_{10};$$

and there are six independent amplitudes.

(3) $P+P\rightarrow P+P(a=P, b=P, c=P, d=P$ or $a=V, b=V, c=V, d=V)$. Time-reversal invariance in channel I leads to $A_{as}=A_{sa}$ and Eq. (2). Since channels II and III are identical, $B_{as}=B_{sa}$ and $C_{\overline{10}}=C_{10}$ and thus also Eqs. (3) and (4) hold. Then Eqs. (2), (3), and (4) lead to R invariance and there are five independent amplitudes.

(4) $P+P\rightarrow V+V(a=P, b=P, c=V, d=V)$. Let $(P_1P_2|V_1V_2)$, $(\bar{V}_1P_1|V_2\bar{P}_2)$, and $(\bar{V}_1P_2|\bar{P}_1V_2)$ be channels I, II, and III. Time-reversal invariance in channel II leads to $B_{as}=B_{sa}$ so that Eq. (3) holds. Also time-reversal invariance in channel III leads to

$$C_d=C_d',\quad C_{as}=C_{sa}',\quad C_{sa}=C_{as}', \quad (5)$$

where C_d stands for the diagonal amplitudes and the prime denotes amplitudes of the form $(PV|VP)$. Next, CP invariance in channel II leads to

$$(V_aP_b|P_cV_d)=(CP(V_aP_b)|CP(P_cV_d))$$
$$=(RE(P_aV_b)|RE(V_cP_d)),$$

where R is the R-conjugation operator and $E|V_aP_b)=|V_bP_a)$. Here V_a is the member of the V octet that corresponds to the member P_a of the P octet. One then obtains $C=C'$ except for $C_{10}=C_{\overline{10}}'$ and $C_{\overline{10}}=C_{10}'$. This is because R operator interchanges $C_{\overline{10}}$ and C_{10}; and both R and E operators change the signs of C_{as} and C_{sa}. Combining these relations with Eq. (5), one has $C_{\overline{10}}=C_{10}$ and $C_{as}=C_{sa}$ so that also Eqs. (2) and (4) hold, and R invariance is satisfied. In other words, for cases (3) and (4) essentially time-reversal invariance

TABLE I. Relations between channel amplitudes.

Process	Relations	Number of independent amplitudes
$PB\to PB$, $VB\to VB$	$A_{as}=A_{sa}$	7
$\bar{B}B\leftrightarrow PP$, $\bar{B}B\leftrightarrow VV$	$B_{\overline{10}}=B_{10}$	7
$BB\to BB$	$A_{as}=A_{sa}=0$	6
$\bar{B}B\to\bar{B}B$	$C_{\overline{10}}=C_{10}$, $C_{as}=C_{sa}$	6
$PP\to PP$, $VV\to VV$	$A_{\overline{10}}=A_{10}$, $A_{as}=A_{sa}=0$	5
$PV\to PV$, $PP\leftrightarrow VV$	(R conjunction invariance)	

and crossing symmetry lead to R invariance. This result holds because the octets P and V contain their charge conjugates. The results are summarized in Table I.

One can use the above results for the channels with nucleon number zero (except for $\bar{B}+B\to P+V$) in order to obtain some spin selection rules in a simple way. Let s be the magnitude of the total intrinsic spin of the system. Then for any state $|ab)$ one has[5]

$$CP|ab)=(-1)^s RE|ab) \quad \text{for } VV \text{ and } PP(s=0) \text{ systems,} \tag{6}$$
$$CP|ab)=(-1)^{s+1}RE|ab) \text{ for the } \bar{B}B \text{ system,}$$

where $E|ab)=|a'b')$, in which $a'(b')$ is the particle corresponding to $b(a)$ in the octet to which $a(b)$ belongs. For example, $E|p_\alpha\bar{\Sigma}_\beta{}^+)=|\Sigma_\alpha{}^-\bar{\Xi}_\beta{}^-)$, where the indices α and β indicate the spin state of each particle. (In particular, for the PP system, the E operation is equivalent to the particle exchange.)

Now CP invariance and Eq. (6) lead to

$$(ab|cd)=\pm(RE(ab)|RE(cd)). \tag{7}$$

The sign to be used depends on the initial and final values of s. In terms of channel amplitudes, the negative sign in Eq. (7) means that $A=0$ except for A_{10} and $A_{\overline{10}}$ for which $A_{10}=-A_{\overline{10}}$. However, as has been shown before, time-reversal invariance in the crossed channel of the processes considered here and the crossing symmetry lead to $A_{10}=A_{\overline{10}}$ so that the case of negative sign in Eq. (7) also implies that $A_{10}=A_{\overline{10}}=0$. Hence all the amplitudes are zero. Therefore, the sign in Eq. (7) should always be positive,[6] and one obtains the spin selection rules listed in Table II. Here one can see, for example, that a nucleon-antinucleon pair can decay into two pseudoscalar mesons only from spin-triplet states.

TABLE II. Spin selection rules.

Process	$\bar{B}B\to\bar{B}B$	$VV\to VV$	$VV\leftrightarrow PP$	$\bar{B}B\leftrightarrow PP$	$\bar{B}B\leftrightarrow VV$
Selection rule[a]	$s_i=s_f$	$s_i+s_f=$even	$s=0, 2$	$s=1$	$s_i+s_f=$odd

[a] The indices i and f denote the initial and the final states, respectively, and s means the magnitude of the total intrinsic spin.

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[5] This follows from the fact that: $C|ab)=R|ab)$ and $P|ab)=(-1)^sE|ab)$ for VV and $PP(s=0)$ systems, $C|ab)=(-1)^{s+L}\times RE|ab)$ and $P|ab)=-(-1)^L|ab)$ for the $B\bar{B}$ system, where L is the magnitude of the orbital angular momentum.

[6] This implies the invariance under the RE operation and leads to $A_{\overline{10}}=A_{10}$, which is consistent with the result obtained before.

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Three-Particle Unitarity Integral*

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Various forms of the production amplitude are proposed which are convenient for the three-particle unitarity integral. Both exact and approximate forms are discussed. It is found that in both cases one can separate the final-state configuration from the over-all kinematics by using discrete variables. Form factors with the three-particle intermediate state are also discussed.

I. INTRODUCTION

ONE of the outstanding difficulties in dispersion theory is the problem of unitarity integral involving more than two particles in the intermediate state.[1] Although many attempts have been made to amend this difficulty, the crude two-particle approximation seems to be the only method giving useful results.[2] In the case of three-particle intermediate state, various authors considered simple Feynman diagrams to study analytic properties of the absorptive part.[3] It

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[1] See, for instance, G. F. Chew, *S-Matrix Theory of Strong Interactions* (W. A. Benjamin, Inc., New York, 1961).

[2] L. F. Cook and B. W. Lee, Phys. Rev. **127**, 283 (1962).

[3] V. N. Gribov and I. T. Dyatlov, Zh. Eksperim. i Teor. Fiz. **42**, 196 (1962); **42**, 1268 (1962) [English transls.: Soviet Phys.—JETP **15**, 140 (1962); **15**, 879 (1962)]; Y. S. Kim, Phys. Rev. **132**, 927 (1963).